

Stability Analysis of DC-DC Converters with Energy Balance Control

Lei Wang, Yang Liu, Yongliang Guo, Qing-Hua Wu, *Life Fellow, IEEE, Fellow, CSEE*,
and Guangfu Tang, *Member, IEEE, Fellow, CSEE*

Abstract—With variation of parameters, DC-DC converters may change from a stable state to an unstable state, which severely degrades the performances of the converter system. In this article, by establishing the state-space average model, the stability and bifurcation of a boost and a buck-boost converter with energy balance control (EBC) is studied, respectively. Then the stability boundary and stable parameter domains are accurately predicted. The obtained stability region provides a parameter regulating range for converter design. Furthermore, compared with the one-cycle control (OCC) method, the EBC possesses an extended stable parameter domain, while avoiding unstable behaviors such as Hopf bifurcation, Quasi-periodic Oscillation even chaos, etc. The theoretic analysis is well validated through simulation and experiment.

Index Terms—DC-DC converter, energy balance control, average model, slow-scale stability.

I. INTRODUCTION

WITH increasing demands in industrial applications, various linear or non-linear control methods such as current-mode control, voltage-mode control, hysteresis control method, model predictive control (MPC), one-cycle control (OCC), model predictive control, bang-bang control and energy balance control (EBC) [1]–[10], have been developed for good dynamic and steady performances. Where the EBC method can achieve one-cycle response since it keeps output energy equal to input energy directly in each switching cycle, which ensures its great disturbance suppression ability to the power source and fast dynamic response to load variations, etc. These advantages make it achieve rapid application in recent years.

A power electronic converter is a highly nonlinear system, so it is prone to exhibit Hopf bifurcation, sub-harmonic oscillation, chaos of many abundant nonlinear behaviors. These nonlinear behaviors lead to instabilities of the converter system. In details, with variation of parameters, an inverter with

the current-mode control exhibits higher periodic solutions and chaos [11]. In a voltage-mode controlled converter, the Neimark-Sacker bifurcation and border collision have been found in the system, which can cause system instability [12]. Due to the advantages of easy implementation and fast load dynamic response, the V^2 control method has been extensively used in converters. However, in [13], [14], it was reported that, once the duty ratio exceeds 0.5, sub-harmonic oscillations occurs, and by building the averaged model, ripple oscillation instability was investigated. In [14], the sub-harmonic oscillations of the buck converters with V^2 and $V^2 I_c$ control were analyzed. Neimark-Sacker bifurcation are proven to exist in the OCC buck-boost converter in [15]. A Cuk converter based on OCC was analyzed and bifurcation was found in the input side of the converter under the variation of some parameters [16]. Studies on a boost converter with the OCC show that the boost converter can be stable in the state of period I in DCM, but in CCM, it can not have stable operation due to low-frequency oscillation [17]. In [18], the transient response characteristic of the three-phase VSCs when the grid voltage dips has been studied, then a serious bifurcation phenomenon was found in the system. The nonlinear dynamics of interconnected power converters has been analyzed in [19]. The nonlinear phenomenon based on a constant power load was analyzed and a sliding mode control method was proposed to ensure stable operation under load variations. Power converters based on a non-ideal power grid have been studied in [20] and bifurcation boundaries were identified. In [21], a DC cascaded system, which consists of two DC-DC converters with average current mode control, has been considered. Slow time scale instability was observed from its discrete-time analysis, computer simulations and experiments. By using bifurcation diagrams, period doubling and subharmonic oscillation were found in the boost converter with constant power loads [22]. These published works show clearly that there exists complex nonlinear behaviors, i.e., Hopf bifurcation, period doubling bifurcation and even chaos in both linear feedback or non-linear feedback controlled converters. Thus, studying the stability to define its stable region is important for the design and application of a novel control method.

The topic of this article is to analyze the stability of the DC-DC converters using the EBC, define its stable parameter domains and provide guidance for industrial applications. The structure is as follows: Sections II and III derive the working principle and average model of an EBC boost converter and an EBC buck-boost converter, then the stability is analyzed based

Manuscript received September August, 2020; revised October 20, 2020; accepted December 4, 2020. Date of online publication June 25, 2021; date of current version March 23, 2023. This work was supported by the State Key Program of National Natural Science Foundation of China (Grant No. U1866210) and National Natural Science Foundation of China (Grant No. 62001169).

L. Wang, Y. Liu, Y. L. Guo, and Q. H. Wu (corresponding author, email: wuqh@scut.edu.cn) are with School of Electric Power Engineering, South China University of Technology, Guangzhou 510641, China.

G. F. Tang is with State Grid Smart Grid Research Institute Co., Ltd., Beijing 102209, China.

DOI: 10.17775/CSEEJPES.2020.03750

on their average models; Section IV carries out the simulation and experimental verification; Section V gives the conclusions.

II. STABILITY ANALYSIS OF AN EBC BOOST CONVERTER

A. The Average Model of an EBC Boost Converter

Figure 1 is a diagram of an EBC boost converter, where u_{in} represents the input voltage, i represents the current of the inductor, u_c represents the voltage across the capacitor, i_o is the current of the load, u is the output voltage and T_s is the switching cycle.

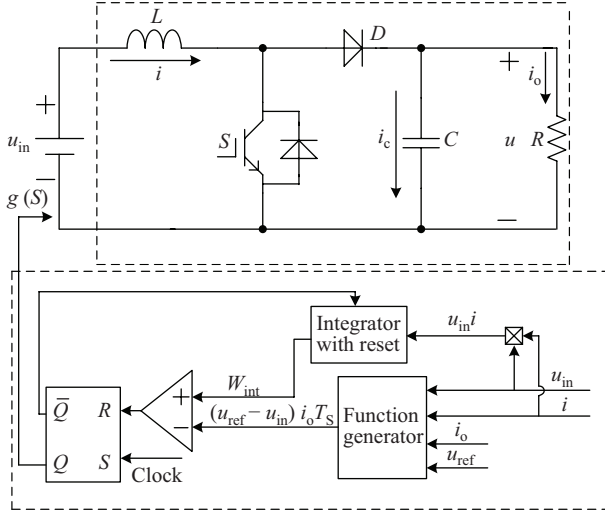


Fig. 1. An EBC boost converter.

The energy balance in the circuit means that energy that was put into the circuit $W_{int}(n)$ equals the sum of the energy the load consumed $W_{out}(n)$, the energy that the reactive components stored $\Delta W_\ell(n)$ and the energy that the circuit loses $W_s(n)$ [10]. Under steady state, since the energy that the inductor absorbs and releases is equal in each switching cycle, $\Delta W_\ell(n)$ can be neglected. Then neglecting the circuit energy losses $W_s(n)$, the control equation of the boost converter can be obtained as follows:

$$\int_{(n-1)T_s}^{(n-1)T_s + t_{on}(n)} u_{in} i dt = (u_{ref} - u_{in}) i_o T_s \quad (1)$$

Then the average model of (1) is obtained as:

$$u_{in} i d = (u_{ref} - u_{in}) i_o \quad (2)$$

where d , which can be expressed as $\frac{t_{on}(n)}{T_s}$, represents the duty cycle.

From (2), d can be derived as:

$$d = \frac{(u_{ref} - u_{in}) i_o(t)}{u_{in} i} = \frac{(u_{ref} - u_{in}) u}{u_{in} i R} \quad (3)$$

As shown in Fig. 1, the EBC is constituted by a RS flip-flop, an comparator and an integrator. During the n^{th} switching period, switch S is turned on by a clock pulse; at this moment, the integration process starts. The output value of the integrator continuously increases and is instantaneously compared with the control reference. Once the integration value reaches the

control reference, the comparator generates a reset pulse, then the output end Q is changed to be 0. The state of S is changed from on to off. Meanwhile, the integrator resets to be zero. S keeps the state of being off until the next clock pulse arrives, at this moment, the $(n+1)^{th}$ switching cycle starts [6], [7].

Such a converter is described as:

$$\begin{aligned} \frac{di}{dt} &= \frac{u_{in}}{L} - \frac{1-s}{L} u \\ \frac{du}{dt} &= -\frac{u}{RC} - \frac{1-s}{C} i \end{aligned} \quad (4)$$

where $s = 0$ denotes that S is in the off-state, $s = 1$ denotes that S is in the on-state.

Replaying s in (4) with d derived in (3), the state-space averaging model of the EBC boost converter is deduced as

$$\begin{aligned} \frac{di}{dt} &= \frac{u_{in}}{L} - \frac{u}{L} \left[1 - \frac{(u_{ref} - u_{in})u}{u_{in} i R} \right] \\ \frac{du}{dt} &= -\frac{u}{RC} - \frac{i}{C} \left[1 - \frac{(u_{ref} - u_{in})u}{u_{in} i R} \right] \end{aligned} \quad (5)$$

B. The Stability Analysis of the EBC Boost Converter

Setting $\frac{di}{dt}$ and $\frac{du}{dt}$ in (5) to be 0, the equilibrium point can be obtained as follows:

$$\begin{aligned} V &= u_{ref}, \\ I &= \frac{u_{ref}^2}{u_{in} R} \end{aligned} \quad (6)$$

At such an equilibrium point, the Jacobian matrix can be obtained as following.

$$\mathbf{J} = \begin{pmatrix} -\frac{u^2(u_{ref} - u_{in})}{Lu_{in} R I^2} & -\frac{1}{L} + \frac{2(u_{ref} - u_{in})u}{u_{in} I R L} \\ \frac{1}{C} & -\frac{1}{RC} - \frac{(V_{ref} - V_{in})}{C u_{in} R} \end{pmatrix} \quad (7)$$

Substituting (6) into the Jacobian matrix gets

$$\mathbf{J} = \begin{pmatrix} -\frac{(u_{ref} - u_{in})u_{in} R}{Lu_{ref}^2} & \frac{u_{ref} - 2u_{in}}{Lu_{ref}} \\ \frac{1}{C} & -\frac{u_{ref}}{C u_{in} R} \end{pmatrix} \quad (8)$$

According to $\det[\lambda \mathbf{I} - \mathbf{J}] = 0$, where $\mathbf{I}$ is a 2×2 unity matrix, the characteristic equation is derived as follows:

$$\begin{aligned} f(\lambda) &= \left(\lambda + \frac{(u_{ref} - u_{in})u_{in} R}{Lu_{ref}^2} \right) \left(\lambda + \frac{u_{ref}}{C u_{in} R} \right) - \frac{u_{ref} - 2u_{in}}{LC u_{ref}} \\ &= \lambda^2 + \left[\frac{(u_{ref} - u_{in})u_{in} R}{Lu_{ref}^2} \right] \lambda + \frac{u_{in}}{LC u_{ref}} \end{aligned} \quad (9)$$

A second-order Padé approximation [23] is used for the stability analysis of the converter. According to the second-order Padé approximation, (9) is rewritten as follows:

$$a_2 \lambda^2 + a_1 \lambda + a_0 \quad (10)$$

where $a_2 = 1$, $a_1 = \frac{(u_{ref} - u_{in})u_{in} R}{Lu_{ref}^2}$ and $a_0 = \frac{u_{in}}{LC u_{ref}}$.

According to Routh criterion [23], all roots are in the left half plane in the stable condition of the system, and for a second-degree polynomial, all a_i are positive is a sufficient and necessary condition to ensure that all roots are located in the left side. Since $u_{ref} > u_{in}$ holds in boost converters, thus

$a_2 > 0$, $a_1 > 0$ and $a_0 > 0$ are obtained. Thus, no matter how parameters change, the sufficient and necessary condition for stability of the EBC boost converter are always met, which shows wide stable parameter domains.

III. STABILITY ANALYSIS OF AN EBC BUCK-BOOST CONVERTER

A. The Working Principle and Average Model of an EBC Buck-boost Converter

Figure 2 is the structure of an EBC buck-boost converter, where u_{in} represents the input voltage, i denotes the current flowing through the inductor, i_o represents the current flowing through the load, u_c represents the voltage across the capacitor, u represents the output voltage and T_s is the switching cycle.

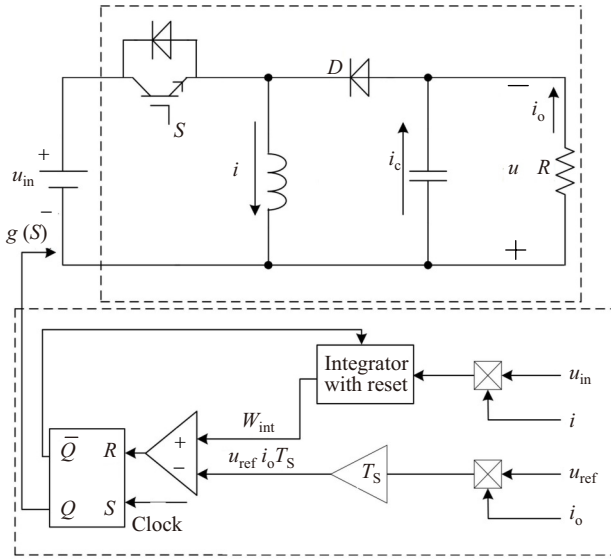


Fig. 2. An EBC buck-boost converter.

Similar to the principle of the EBC boost converter, under the steady state, the energy balance in such circuit can be written as:

$$\int_{(n-1)T_s}^{(n-1)T_s + t_{on}(n)} u_{in} i dt = u_{ref} i_o T_s \quad (11)$$

Then the average model of (11) is obtained as:

$$u_{in} i d = u_{ref} i_o \quad (12)$$

where d , which can be expressed as $\frac{t_{on}(n)}{T_s}$, is the duty ratio.

From (12), d can be derived as:

$$d = \frac{u_{ref} i_o(t)}{u_{in} i} = \frac{u_{ref} u}{u_{in} i R} \quad (13)$$

Similar to the implementation of the EBC boost converter, during the n^{th} switching period, a clock pulse turns on switch S; at this moment, the integration process starts. The output value of the integrator W_{int} continuously increases from its initial value and is instantaneously compared with the control reference. Once the integration value reaches the control reference, the comparator generates a reset pulse, then the output end Q is changed to be 0. The state of S is changed

from on to off. Meanwhile, the integrator resets to be zero. S keeps the state of being off until the next clock pulse arrives; at this moment, the $(n+1)^{\text{th}}$ switching cycle starts [6] [7].

Such a converter is described as:

$$\begin{aligned} \frac{di}{dt} &= -\frac{u_{in}}{L}s - \frac{u}{L}(1-s), \\ \frac{du}{dt} &= -\frac{u}{RC}s + (1-s)\left(\frac{i}{C} - \frac{u}{RC}\right) \end{aligned} \quad (14)$$

where $s = 1$ represents that S_1 is in the on-state, $s = 0$ represents that S_1 is in the off-state.

Replaying s in (14) with d derived in (13), the average state space model of the EBC buck converter is deduced as follows.

$$\begin{aligned} \frac{di}{dt} &= -\frac{u_{ref} u}{LR} + \frac{u_{ref} u^2}{Lu_{in} R}, \\ \frac{du}{dt} &= \frac{i}{C} - \frac{u}{RC} - \frac{u_{ref} u}{CRu_{in}} \end{aligned} \quad (15)$$

B. The Stability Analysis of the EBC Buck-boost Converter

Setting $\frac{di}{dt}$ and $\frac{du}{dt}$ in (15) to be 0, the equilibrium point can be obtained as follows:

$$\begin{aligned} V &= u_{ref}, \\ I &= \frac{u_{ref}(u_{in} + u_{ref})}{Ru_{in}} \end{aligned} \quad (16)$$

At such an equilibrium point, the Jacobian matrix is obtained as follows:

$$J = \begin{pmatrix} -\frac{(u_{in} + V)Vu_{ref}}{LRu_{in}I^2} & \frac{u_{ref}}{LR} + \frac{2u_{ref}V}{LRu_{in}} \\ \frac{1}{C} & -\frac{1}{RC} - \frac{u_{ref}}{RCu_{in}} \end{pmatrix} \quad (17)$$

Substituting (16) into (17) gets

$$J = \begin{pmatrix} -\frac{Ru_{in}}{L(u_{in} + u_{ref})} & \frac{u_{ref}}{L(u_{in} + u_{ref})} \\ \frac{1}{C} & -\frac{u_{in} + u_{ref}}{CRu_{in}} \end{pmatrix} \quad (18)$$

According to $\det[\lambda I - J] = 0$ and the second-order Padé approximation, the characteristic quasi-polynomial equation can be written as follows:

$$a_2 \lambda^2 + a_1 \lambda + a_0 \quad (19)$$

where $a_2 = 1$, $a_1 = \frac{Ru_{in}}{L(u_{in} + u_{ref})} + \frac{(u_{in} + u_{ref})}{RCu_{in}}$ and $a_0 = \frac{u_{in}}{LC(u_{in} + u_{ref})}$.

From (19), it is obvious that $a_2 > 0$, $a_1 > 0$ and $a_0 > 0$. Similar to the boost converter, with the EBC, the buck-boost converter also has an extended stable parameter domain.

IV. SIMULATION AND EXPERIMENTAL VERIFICATIONS

To verify the theoretical analysis of the stability of the EBC converter in the previous section, taking u_{ref} as the example, the dynamic stability of the converters under varying u_{ref} is tested in simulation and experiment. To evaluate the advantages of the EBC with wider stable parameter domains, a comparative study to the one-cycle control (OCC) is presented. Table I lists the circuit parameters of the converters. The characteristic roots of the EBC and OCC DC-DC converter

TABLE I
THE CIRCUIT PARAMETERS

Description	u_{in} (V)	L (mH)	C (μ F)	R (Ω)	f_s (Hz)
Boost converter	5	3	460	30	5000
Buck-boost converter	5	15	470	50	5000

based on these circuit parameters listed in Table I can be calculated respectively. From Table II, it can be observed that, when $u_{ref} \leq 10$ V, the real part of the characteristic roots of the OCC converter are negative, which means these roots are located in the left-half plane, exactly as shown in Fig. 3(a). Apparently, the converter is stable. However, when $u_{ref} \geq 10$ V, the characteristic roots of the OCC converter change to be positive real parts, thus the roots move gradually toward the right-half plane. From Table II, it can be seen the eigenvalues of the Jacobian matrix of the OCC boost converter, thus the criteria is as follows [24]:

- 1) If the real parts of the eigenvalues of the Jacobian matrix are negative, the converter operates stably.
- 2) If the real parts of the pair of complex conjugate eigenvalues change to be positive, which means that the eigenvalues move to the right-half plane, the Hopf bifurcation occurs.

Thus when $u_{ref} \leq 10$ V, the OCC boost converter losses its stability and exhibits Hopf bifurcation.

TABLE II
THE CHARACTERISTIC ROOTS OF THE BOOST CONVERTER
UNDER VARIED u_{ref}

u_{ref}	EBC	OCC
7 V	$\lambda_{1,2} = -102.04 \pm 712.17i$	$\lambda_{1,2} = -21.74 \pm 719.12i$
8 V	$\lambda_{1,2} = -117.1875 \pm 662.7i$	$\lambda_{1,2} = -14.49 \pm 672.82i$
9 V	$\lambda_{1,2} = -123.46 \pm 662.36i$	$\lambda_{1,2} = -7.25 \pm 634.45i$
10 V	$\lambda_{1,2} = -125.00 \pm 588.81i$	$\lambda_{1,2} = 0 \pm 601.93i$
11 V	$\lambda_{1,2} = -123.97 \pm 560.37i$	$\lambda_{1,2} = 7.25 \pm 573.87i$
12 V	$\lambda_{1,2} = -121.53 \pm 535.87i$	$\lambda_{1,2} = 14.49 \pm 549.29i$
13 V	$\lambda_{1,2} = -118.34 \pm 514.49i$	$\lambda_{1,2} = 21.74 \pm 527.48i$
...	...	...
25 V	$\lambda_{1,2} = -80 \pm 372.19i$	$\lambda_{1,2} = 108.68 \pm 364.85i$
...	...	...
30 V	$\lambda_{1,2} = -69.44 \pm 340.52i$	$\lambda_{1,2} = 144.90 \pm 315.87i$

In contrast, Table II provides the characteristic roots of the EBC boost converter under varying u_{ref} . The calculated results in Table II show that, when $u_{ref} = 7$ V, the real part of the eigenvalues are negative, and with the increasing of the value of u_{ref} , the eigenvalues always have negative real part. Even when u_{ref} increases to 30 V, the eigenvalue still have negative real part. Such results illustrate the roots are always located in the left-half side, as shown in Fig. 3(b), which illustrates better stability of the converter with the EBC.

Such comparisons can be observed from the bifurcation diagrams in Fig. 4. In Fig. 4, the bifurcation diagrams are

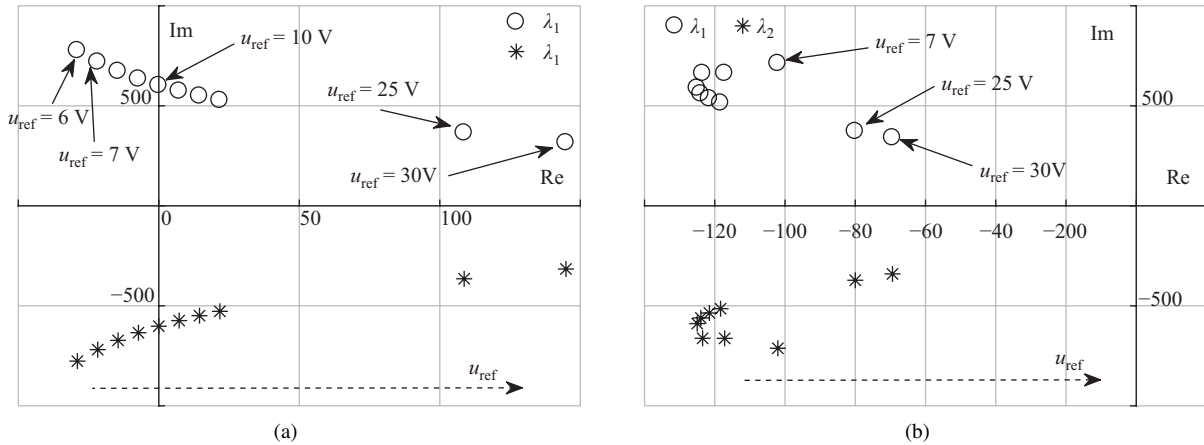


Fig. 3. The characteristic roots of the boost converter with variation of u_{ref} . (a) OCC; (b) EBC.

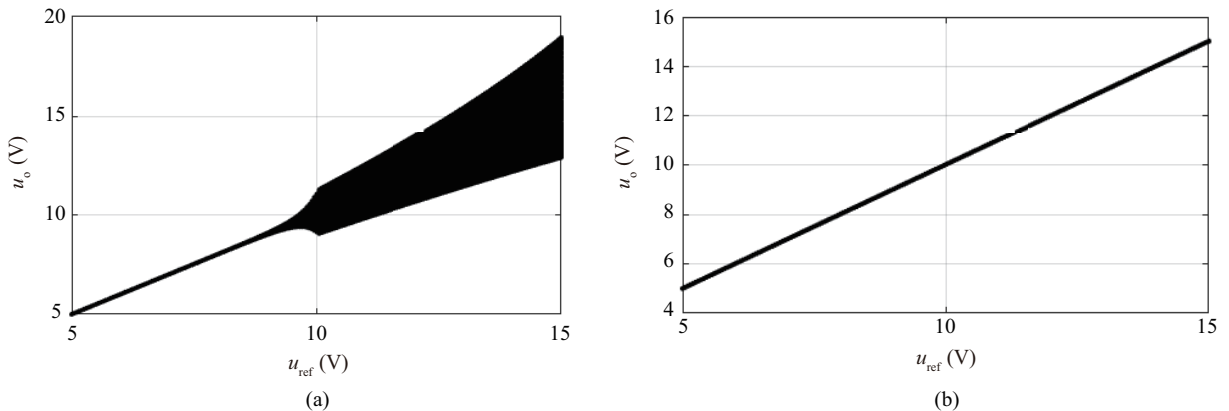


Fig. 4. The bifurcation diagram of the boost converter with variation of u_{ref} . (a) OCC; (b) EBC.

developed where u_{ref} varies from 5 V to 15 V. It is found from Fig. 4(a) that the system operates stably when u_{ref} is less than 10 V, however, when u_{ref} is greater than 10 V, OCC boost converter losses stability and exhibits oscillations. In contrast, from Fig. 4(b), it can be found, even when u_{ref} increases to 15 V, the EBC boost converter remain stable throughout.

Table III shows the calculated characteristic roots of the OCC and EBC buck-boost converter. As shown in Table III, when $u_{\text{ref}} \leq 8$ V, the characteristic roots of the converter using the OCC possess negative real parts, which are located in the left-half side, as shown in Fig. 5(a). In these states, the system can operate stably. However, the real parts change from negative to positive when u_{ref} increases to 9 V. Thus the roots move to the right-half side. In these states, the system loses stable operation with the Hopf bifurcation or other non-linear phenomenons occurring.

In contrast, from Table III, it is observed that, with the varying of u_{ref} , the characteristic roots always have negative real parts. Therefore, the roots are always located in the left-half side, as shown in Fig. 5(b). Such results illustrate the converter is stable even though u_{ref} varies widely.

Figure 6 indicates the Bifurcation diagrams, from which such comparisons can also be observed. Fig. 6(a) illustrates the system is kept stable when u_{ref} is less than 8.1 V, however, when u_{ref} is greater than 8.1 V, the OCC buck-boost converter

TABLE III
THE CHARACTERISTIC ROOTS OF THE BUCK-BOOST CONVERTER
UNDER VARIED u_{ref}

u_{ref}	EBC	OCC
4 V	$\lambda_1 = -1886.6;$ $\lambda_2 = -41.77$	$\lambda_{1,2} = -13.71 \pm 280.38i$
5 V	$\lambda_1 = -1710.30;$ $\lambda_2 = -41.47$	$\lambda_{1,2} = -10.64 \pm 266.09i$
6 V	$\lambda_1 = -1567.63;$ $\lambda_2 = -41.12$	$\lambda_{1,2} = -7.35 \pm 253.81i$
7 V	$\lambda_1 = -1450.26;$ $\lambda_2 = -40.75$	$\lambda_{1,2} = -3.90 \pm 243.08i$
8 V	$\lambda_1 = -1352.34;$ $\lambda_2 = -40.34$	$\lambda_{1,2} = -0.33 \pm 233.57i$
9 V	$\lambda_1 = -1269.70;$ $\lambda_2 = -39.9$	$\lambda_{1,2} = 3.34 \pm 225.05i$
10 V	$\lambda_1 = -1199.34;$ $\lambda_2 = -39.42$	$\lambda_{1,2} = 7.09 \pm 217.33i$
11 V	$\lambda_1 = -1138.90;$ $\lambda_2 = -38.90$	$\lambda_{1,2} = 10.90 \pm 210.26i$
12 V	$\lambda_1 = -1086.67;$ $\lambda_2 = -38.39$	$\lambda_{1,2} = 14.77 \pm 203.72i$

oscillates. In contrast, from Fig. 6(b), it can be found, even when u_{ref} increases to 10 V, the EBC buck-boost converter remain stable throughout.

A. Simulation Results of the Boost and Buck-boost Converter

Figures 7–10 display the simulation results of the EBC and

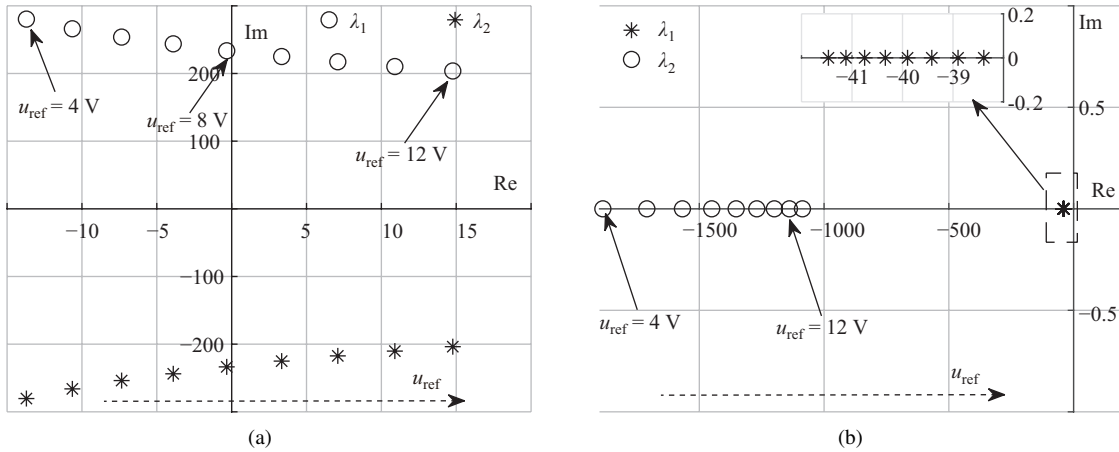


Fig. 5. The characteristic roots of the buck-boost converter with variation of u_{ref} . (a) OCC; (b) EBC.

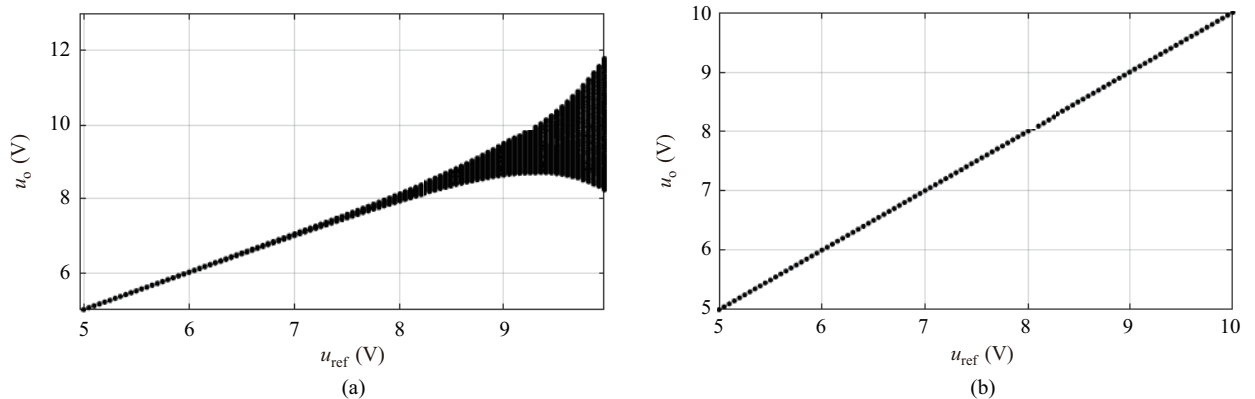


Fig. 6. The bifurcation diagram of the buck-boost converter with variation of u_{ref} . (a) OCC; (b) EBC.

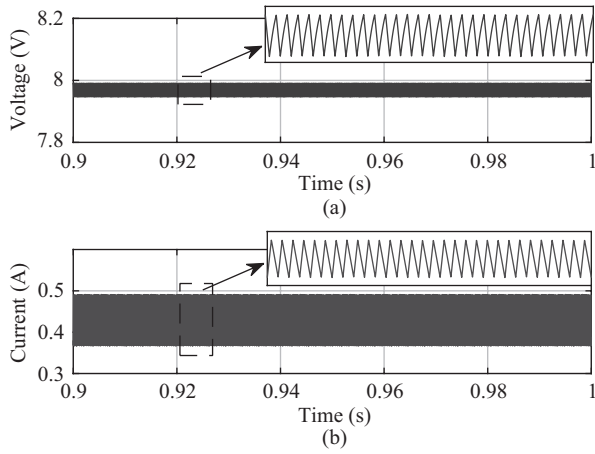


Fig. 7. Results of simulation of the EBC boost converter under $u_{\text{ref}} = 8$ V. (a) u ; (b) i .

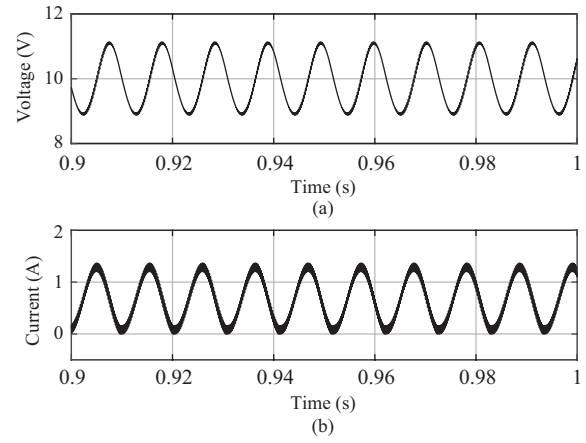


Fig. 10. Results of simulation of the OCC boost converter under $u_{\text{ref}} = 10$ V. (a) u ; (b) i .

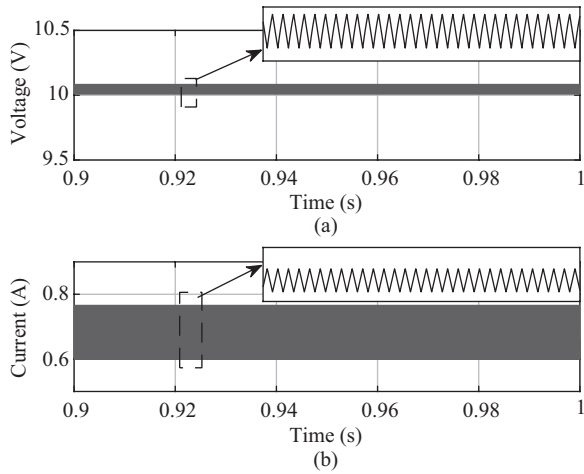


Fig. 8. Results of simulation of the EBC boost converter under $u_{\text{ref}} = 10$ V. (a) u ; (b) i .

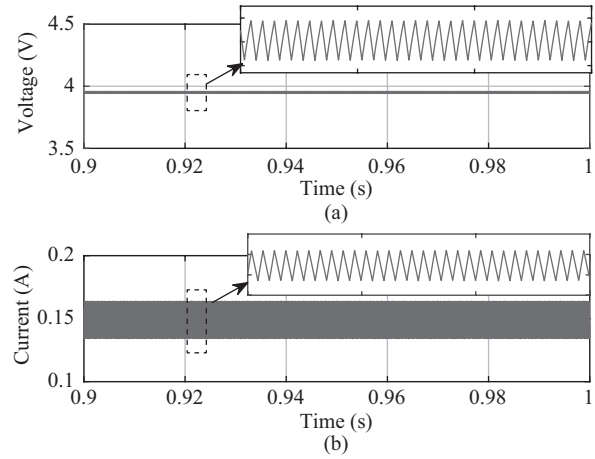


Fig. 11. Results of simulation of the EBC buck-boost converter under $u_{\text{ref}} = 4$ V. (a) u ; (b) i .

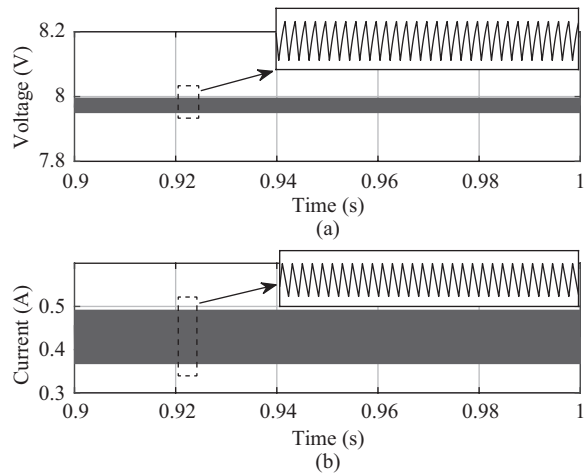


Fig. 9. Results of simulation of the OCC boost converter under $u_{\text{ref}} = 8$ V. (a) u ; (b) i .

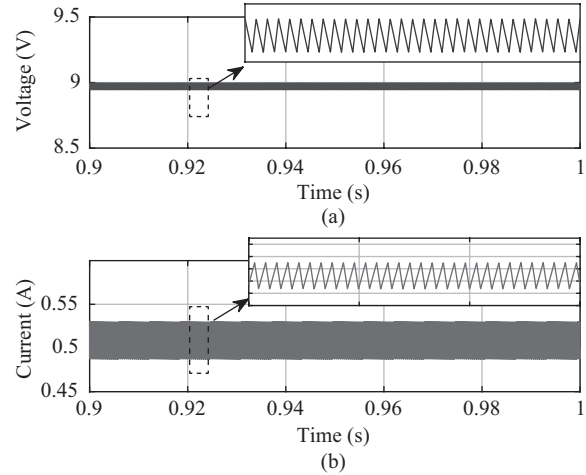


Fig. 12. Results of simulation of the EBC buck-boost converter under $u_{\text{ref}} = 9$ V. (a) u ; (b) i .

OCC boost converter. From Fig. 9, it is observed that, under $u_{\text{ref}} = 8$ V, the converter kept steady operation with no non-linear phenomena occurring. But when u_{ref} increases to 10 V, which is grater than $2 * u_{\text{in}}$, the inductor current exhibits Hopf

bifurcation, leading to oscillations of the output voltage, as shown in Fig. 10. Obviously, the converter loses stability and works in a continuous oscillation mode. The bifurcations will seriously affect performance of the system.

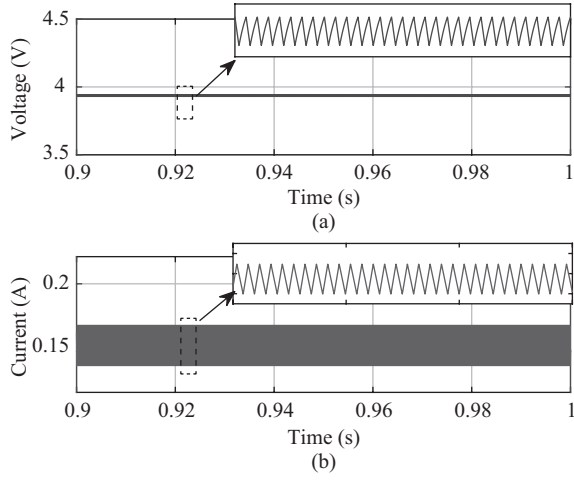


Fig. 13. Results of simulation of the OCC buck-boost converter under $u_{\text{ref}} = 4$ V. (a) u ; (b) i .

In contrast, Figs. 7 and 8 display the simulation results of an EBC boost converter. From the results, it is observed that, under $u_{\text{ref}} = 8$ V and $u_{\text{ref}} = 10$ V, no non-linear phenomena occur, which means the converter can maintain a stable state under these parameters. Obviously, with the varying of u_{ref} , the converter can always keep stable operation.

Figures 11–14 show the simulation results of the EBC and OCC buck-boost converter. From Fig. 11, it can be observed that, under $u_{\text{ref}} = 4$ V, no non-linear phenomena occur, thus, the converter can operate stably. From Fig. 14, it is observed that, when u_{ref} is increased to 9 V, the system loses stable operation, Hopf bifurcation occurs. If u_{ref} is continually increased, other non-linear phenomena will occur. In contrast, with the EBC, there is no bifurcation and other non-linear

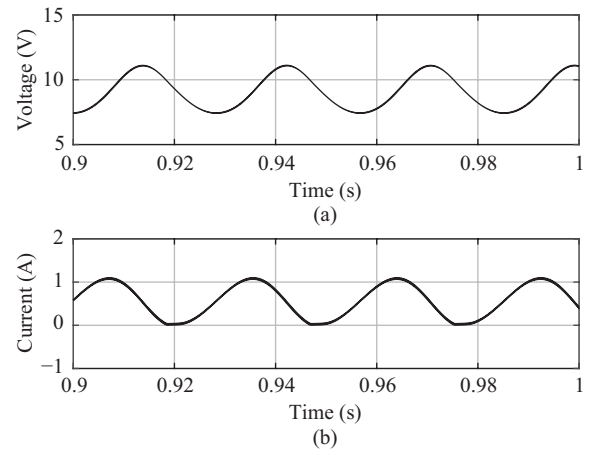


Fig. 14. Results of simulation of the OCC buck-boost converter under $u_{\text{ref}} = 9$ V. (a) u ; (b) i .

phenomena occur under $u_{\text{ref}} = 4$ V and $u_{\text{ref}} = 9$ V. As shown in Figs. 11 and 12, the converter with the EBC can keep stable operation.

B. Experimental Results of the Boost and Buck-boost Converter

According to the parameters listed in Table I, the experimental prototypes are built respectively. In the experiment, a Mosfet IRF640N and fast recovery diode BYW29-200 are used. HALL sensors are chosen for measuring the voltage and current. The control part of the experimental platform is implemented by StarSim Control Block. Figs. 15 and 16 show the experimental results. It is observed from Fig. 15 that the experimental results confirm the simulation results of Figs. 7–10. With the OCC, under $u_{\text{ref}} = 8$ V, the operation

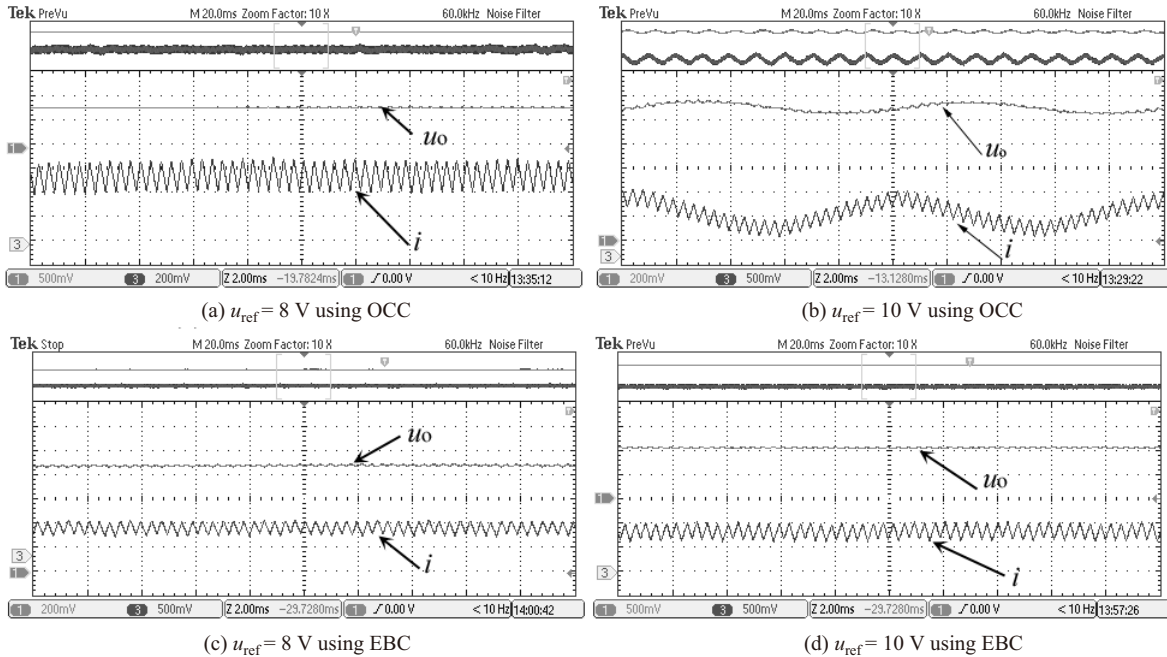


Fig. 15. Results of experiment of the boost converter with variation of u_{ref} .

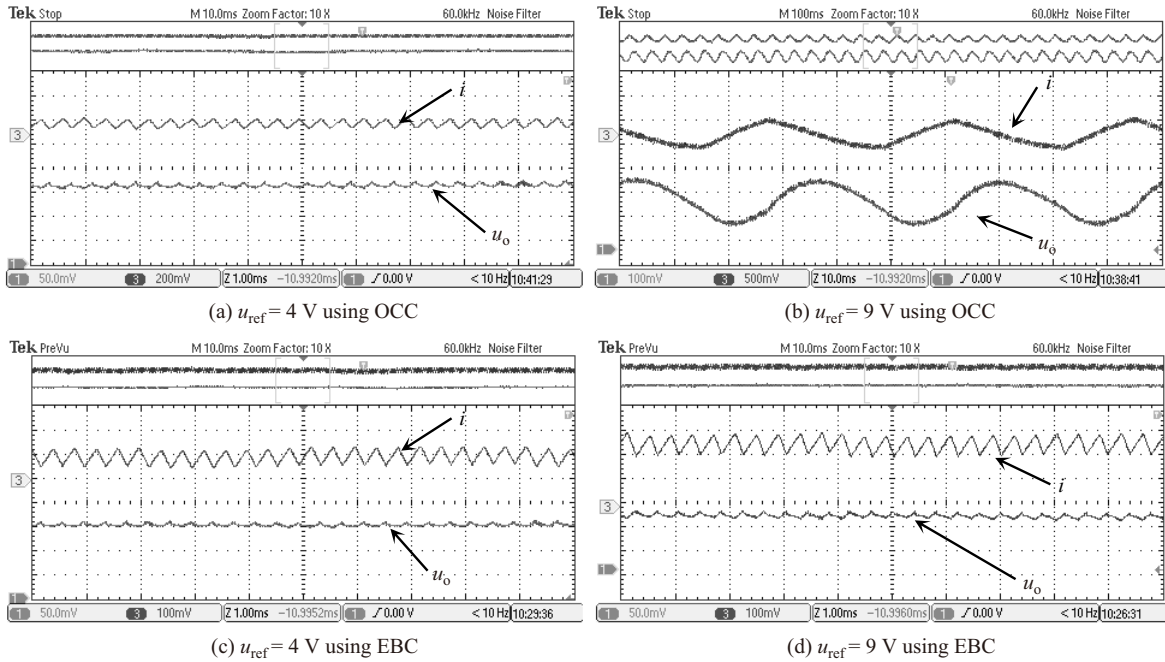


Fig. 16. Results of experiment of the buck-boost converter with variation of u_{ref} .

of the converter is stable and no bifurcation or other non-linear phenomena occur. But the converter losses stability when u_{ref} is increased to 10 V, the inductor current exhibits oscillations due to the Hopf bifurcation, which leads to low-frequency oscillations of the output voltage. In contrast, with the EBC, under $u_{ref} = 8$ V, no non-linear phenomena occur, the converter operate stably. And when u_{ref} is increased to 10 V, the converter can still operate stably. This means that converters using the EBC produce more excellent dynamic stability than converters using the OCC.

The results in Fig. 16 show the experimental results, which confirm the simulation results in Figs. 11–14. The results illustrate that, when $u_{ref} = 4$ V, no non-linear phenomena such as bifurcation is found in the OCC buck-boost converter. However, with the increasing of u_{ref} , the converter losses its stability. The inductor current exhibits oscillations, as illustrated in Fig. 16(b). In contrast, from Fig. 16(c) and (d), it is observed that, with the EBC, when $u_{ref} = 4$ V and $u_{ref} = 9$ V, the converter can keep stable operation, no Hopf bifurcation or other non-linear phenomena appear.

Thus, the simulation and experiment results are found to be mostly in agreement with the presented theories, whereby the EBC DC-DC converter system has more excellent dynamic stability, which lies in wider stable parameter domains.

V. CONCLUSION

Taking a boost and a buck-boost converter as examples, the dynamic stability and bifurcation of DC-DC converters controlled by the EBC have been analyzed in this paper. At first, the state-space averaged models have been built. Then the stability boundary has been found and stable parameter domains has been obtained by analyzing its Jacobian matrix. The analysis results demonstrate the characteristic roots of

the converter with the EBC are always located in the left half plane, thus, the converter with the EBC has wider stable parameter domains, which is reflected in no bifurcation or other non-linear phenomena occurring with variation of the circuit parameters. Take the reference output voltage for example, simulation and experiment results are consistent with the presented theories, which demonstrate that the EBC boost and buck-boost converter can keep stable operation under the varying of u_{ref} . No bifurcation or other slow-scale oscillation phenomena occurs under change of the reference voltage. The comparison with that of the OCC DC-DC converters illustrates the EBC DC-DC converter system has more excellent dynamic stability since it can maintain stable operation in a wider domain of parameter ranges. Results reveal the advantage of the EBC in non-linear stability while promoting development of non-linear science study toward new control methods.

REFERENCES

- [1] L. P. Guo, J. Y. Hung, and R. M. Nelms, "Evaluation of DSP-based PID and fuzzy controllers for DC-DC converters," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 6, pp. 2270–2248, Jun. 2009.
- [2] V. P. Arikatla and J. A. A. Qahouq, "Adaptive digital proportional-integral-derivative controller for power converters," *IET Power Electronics*, vol. 5, no. 3, pp. 341–348, Mar. 2012.
- [3] P. J. Liu, Y. M. Lai, P. C. Lee, and H. S. Chen, "Fast-transient DC-DC converter with hysteresis prediction voltage control," *IET Power Electronics*, vol. 10, no. 3, pp. 271–278, Mar. 2017.
- [4] R. J. Wai and L. C. Shih, "Design of voltage tracking control for DC-DC boost converter via total sliding-mode technique," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 6, pp. 2502–2511, Jun. 2011.
- [5] K. M. Smedley and S. Cuk, "One-cycle control of switching converters," *IEEE Transactions on Power Electronics*, vol. 10, no. 6, pp. 625–633, Nov. 1995.
- [6] L. Wang, Q. H. Wu, W. H. Tang, B. Li, and Z. X. Xu, "Energy balance analysis and control for boost converters," in *43rd Annual Conference of the IEEE Industrial Electronics Society*, 2017, pp. 8194–8200.

- [7] L. Wang, Q. H. Wu, Y. K. Tao, and W. H. Tang, "Switching control of buck converter based on energy conservation principle," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 5, pp. 1779–1787, Sep. 2016.
- [8] M. T. Touati, S. Li, J. Wu, M. Ali and M. M. Khan, "Satisfactory model predictive control for a hybrid single-phase seven-level converter," in *CSEE Journal of Power and Energy Systems*, vol. 7, no. 5, pp. 1102–1112, Sept. 2021.
- [9] Y. Liu, K. Xiahou, Q. H. Wu, H. Pang and G. Tang, "Robust bang-bang control of disturbed nonlinear systems based on nonlinear observers," in *CSEE Journal of Power and Energy Systems*, vol. 6, no. 1, pp. 193–202, March 2020.
- [10] L. Wang, Q. H. Wu, and W. H. Tang, "Improved energy balance control for boost converters without estimating circuit energy losses," *IEEE Access*, vol. 8, pp. 146323–146330, Jun. 2020.
- [11] H. H. C. Iu and B. Robert, "Control of chaos in a PWM current-mode H-bridge inverter using time-delayed feedback," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 50, no. 8, pp. 1125–1129, Aug. 2003.
- [12] F. Xie, B. Zhang, and D. Y. Qiu, "Stabilizing the nonlinear dynamic behavior of LLC resonance full-bridge DC-DC converter under voltage mode control," in *2014 International Power Electronics and Application Conference and Exposition*, 2014, pp. 1093–1097.
- [13] G. H. Zhou, J. P. Xu, Y. Y. Jin, and J. Sha, "Stability analysis of V^2 controlled buck converter operating in CCM and DCM," in *2010 International Conference on Communications, Circuits and Systems (ICCCAS)*, 2010, pp. 551–555.
- [14] J. Sun, "Characterization and performance comparison of ripple-based control for voltage regulator modules," *IEEE Transactions on Power Electronics*, vol. 21, no. 2, pp. 346–353, Mar. 2006.
- [15] Y. L. Guo, Q. H. Wu, L. Wang, and G. F. Tang, "Stability enhancement of one-cycle controlled buck-boost converters with a composite function embedded," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 67, no. 10, pp. 3512–3520, Oct. 2020.
- [16] W. Ma, L. Wang, R. Zhang, J. H. Li, Z. M. Dong, Y. H. Zhang, M. Hu, and S. X. Liu, "Hopf bifurcation and its control in the one-cycle controlled Cuk converter," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 66, no. 8, pp. 1411–1415, Aug. 2019.
- [17] J. P. Wang, L. K. Hou, L. Zhang, S. Xiang, and Y. G. He, "Analysis of the low-frequency oscillation phenomenon in constant-on-time controlled boost converter," in *2015 IEEE Energy Conversion Congress and Exposition (ECCE)*, 2015, pp. 1571–1574.
- [18] M. Huang, Y. Peng, C. K. Tse, Y. S. Liu, J. J. Sun, and X. Zha, "Bifurcation and large-signal stability analysis of three-phase voltage source converter under grid voltage dips," *IEEE Transactions on Power Electronics*, vol. 32, no. 11, pp. 8868–8879, Nov. 2017.
- [19] L. Benadero, R. Cristiano, D. J. Pagano, and E. Ponce, "Nonlinear analysis of interconnected power converters: a case study," *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, vol. 5, no. 3, pp. 326–335, Sep. 2015.
- [20] C. Wan, M. Huang, C. K. Tse, and X. B. Ruan, "Stability of interacting grid-connected power converters," *Journal of Modern Power Systems and Clean Energy*, vol. 1, no. 3, pp. 245–253, Dec. 2013.
- [21] L. Ding, S. C. Wong, and C. K. Tse, "Bifurcation analysis of a current-mode-controlled DC cascaded system and applications to design," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 8, no. 4, pp. 3214–3224, Dec. 2020.
- [22] A. El Aroudi, R. Haroun, M. S. Al-Numay, J. Calvente, and R. Giral, "Fast-scale stability analysis of a DC-DC boost converter with a constant power load," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 9, no. 1, pp. 549–558, Feb. 2021.
- [23] K. Ogata, "Modern control engineering," in *Prentice-Hall: Englewood Cliffs*, NJ, 2001.
- [24] S. Banerjee and G. C. Verghese, *Nonlinear Phenomena in Power Electronics: Bifurcations, Chaos, Control, and Applications*, Hoboken: Wiley, 2001.



Lei Wang received the B.Eng. and M.Sc. degrees in Electrical Engineering from Chongqing University, Chongqing, China, in 2008 and 2011, respectively, and the Ph.D. degree in Electrical Engineering from the South China University of Technology, Guangzhou, China, in 2016. She is currently with Guangzhou Maritime University. Her research interests include renewable energy integration in power grids, modeling and control of power converters, and nonlinear analysis of power electronics.



Yang Liu received a B.E. degree and a Ph.D degree in Electrical Engineering from South China University of Technology (SCUT), Guangzhou, China, in 2012 and 2017, respectively. He is currently an Associate Professor in the School of Electric Power Engineering, SCUT. His research interests include the areas of power system stability analysis and control, control of wind power generation systems, and nonlinear control theory. He has authored or co-authored more than 30 peer-reviewed SCI journal papers.



Yongliang Guo received the B.Eng. degree in Electrical Engineering from Zhejiang University, Hangzhou, China, in 2017. He is currently pursuing the master's degree with the School of Electric Power Engineering, South China University of Technology. His research interests include modeling and control of power converters, nonlinear analysis of power electronics, and fault diagnosis of converters.



Qing-Hua Wu received an M.Sc. degree in Electrical Engineering from Huazhong University of Science and Technology, Wuhan, China, in 1981, and a Ph.D. degree in Electrical Engineering from the Queen's University of Belfast (QUB), Belfast, U.K., in 1987. He worked as the Chair Professor at Liverpool University from 1995 to 2012. He is currently a Distinguished Professor and the Director of Energy Research Institute, South China University of Technology, Guangzhou, China. He is a Life Fellow of IEEE, Fellow of IET, CSEE Fellow, Fellow of AAIA, and Chartered Engineer. He has authored and coauthored more than 350 journal papers, 20 book chapters and 4 research monographs published by Springer Nature. His research interests include nonlinear adaptive control, mathematical morphology, artificial intelligence, and power system control and operation.



Guangfu Tang received the B.S. degree from Xi'an Jiaotong University, Xi'an, China, in 1990, and the M.S. and Ph.D. degrees from the Institute of Plasma Physics, The Chinese Academy of Sciences, Hefei, China, in 1993 and 1996, respectively, all in Electrical Engineering. He is currently an Academician with the Chinese Academy of Engineering and the Director of Huairou Laboratory. His research interests include flexible AC transmission systems, the converter valve of high voltage and high voltage in DC transmission systems, and the voltage-source converter-based high-voltage DC transmission systems, and DC grid. Dr. Tang was a regular Member of CIGRE SC B4 committee before. He is a Member of CIGRE SC B4 AG4 "HVDC System Performance". And he was the Convener of CIGRE SC B4 Working Group B4-48 "Components Testing of VSC System for HVDC Applications." He also is a Member of IEC SC22F WG25 and MT22.